ACTA CHEMICA IASI, **25_2**, 145-162 (2017)
DOI: 10.1515/achi-2017-0012

STUDY OF DENDRIMERS BY TOPOLOGICAL INDICES

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Abstract: In this paper, five degree based topological indices, the first Zagreb (M_1), second Zagreb (M_2), first multiple Zagreb (PM_1), second multiple Zagreb (PM_2), and the hyper Zagreb (HM) indices of two types of dendrimers are studied. In addition, two distance based topological indices, the total eccentricity (θ) and eccentric connectivity (ξ^c) indices of these dendrimers are computed.

***Keywords*:** Dendrimer; Vertex-degree; Distance; Polynomial; Topological index.

Introduction

In 1978 Dendrimer chemistry was first introduced by Vögtle et al.[1] Dendrimers find applications in many diverse fields including drug delivery, biology, gene therapy, nanotechnology, photonics, etc.[2-4]. Dendrimers are large and complex molecules with very well-defined chemical structures. At the centre of every dendrimer is its core, radiating outwards are the backbone units and finally the surface groups at the periphery. Topological indices (TIs) are numerical graph invariants that quantitatively characterize

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molecular structure. Topological indices were successfully employed in developing a suitable correlation between chemical structure and biological activity. We encourage reader to references[5-14] to study some of topological indices of various graph families, dendrimers and nanostructures.

The present paper is organized as follows. In the second part of this work, we give the necessary definitions while the third section gives the main results; namely we determine Zagreb polynomials, first Zagreb index, second Zagreb index, first multiple Zagreb index, second multiple Zagreb index, hyper-Zagreb index, total eccentricity polynomial, eccentric connectivity polynomial, total eccentricity index and the eccentric connectivity index for two types of dendrimers. Conclusions and references will close the article. All over this paper, our notation is standard and taken mainly from the standard books as like as[15,16].

At the beginning, we first introduce some notations in graph theory, which will be used in the following discussion. A molecular graph is a collection of points representing the atoms in the molecule and set of lines representing the covalent bonds. These points are named vertices and the lines are named edges in graph theory language. Let G be a simple connected graph with vertex set $V(G)$ and edge set $E(G)$. The number of vertices and edges in a graph will be defined by $|V(G)|$ and $|E(G)|$ respectively. For a graph G, we let d_u be the degree of a vertex u in G and $d(u,v)$ be the distance between two vertices u and v in G. For a vertex u of $V(G)$ its eccentricity $\varepsilon_G(u)$ is the largest distance between u and any other vertex v of G, $\varepsilon_G(u) = Max\{d(u,v) | v \in V(G)\}$.

Numerous graph polynomials were introduced in the literature, several of them turned out to be applicable in mathematical chemistry. For instance, Zagreb polynomials, total eccentricity polynomial, eccentric connectivity polynomial, and many others[17-19].

For a (molecular) graph G, the first Zagreb polynomial $M_1(G,x)$ and second Zagreb polynomial $M_2(G,x)$ of a graph G are defined as follows:

$$M_1(G,x) = \sum_{uv\in E(G)} x^{(d_u+d_v)}, \qquad (1)$$

$$M_2(G,x) = \sum_{uv\in E(G)} x^{(d_u\times d_v)}, \qquad (2)$$

where x is a dummy variable.

The first and second Zagreb (M_1 and M_2) indices have been introduced by Gutman and Trinajstić [20] as follows:

$$M_1(G) = \sum_{uv\in E(G)} (d_u + d_v), \qquad (3)$$

$$M_2(G) = \sum_{uv\in E(G)} (d_u \times d_v). \qquad (4)$$

The multiplicative version of these Zagreb indices the first multiple Zagreb index $PM_1(G)$ and the second multiple Zagreb index $PM_2(G)$ of a (molecular) graph G were introduced by Gutman[21] and Ghorbani and Azimi[22] as follows:

$$PM_1(G) = \prod_{uv\in E(G)} (d_u + d_v), \qquad (5)$$

$$PM_2(G) = \prod_{uv\in E(G)} (d_u \times d_v), \qquad (6)$$

Recently, Shirdel et al.[23] introduced hyper-Zagreb index (*HM*). It is defined as follows:

$$HM(G) = \sum_{uv \in E(G)} (d_u + d_v)^2. \quad \textbf{(7)}$$

The total eccentricity polynomial $\theta(G, x)$ and eccentric connectivity polynomial $\xi^c(G, x)$ of G, are defined as follows:

$$\theta(G, x) = \sum_{u \in V(G)} x^{\varepsilon_G(u)}, \quad \textbf{(8)}$$

$$\xi^c(G, x) = \sum_{u \in V(G)} d_u \, x^{\varepsilon_G(u)}. \quad \textbf{(9)}$$

It is easy to see that the total eccentricity index and the eccentric connectivity index of a graph can be obtained from the corresponding polynomials by evaluating their first derivatives at $x = 1$. The total eccentricity and eccentric connectivity indices of G are defined as follows:

$$\theta(G) = \sum_{u \in V(G)} \varepsilon_G(u), \quad \textbf{(10)}$$

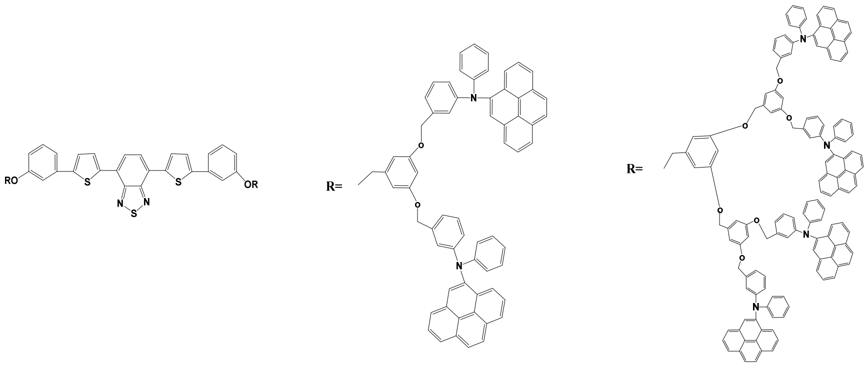
$$\xi^c(G) = \sum_{u \in V(G)} d_u \, \varepsilon_G(u), \quad \textbf{(11)}$$

respectively[24-28]. Eccentric connectivity index has been found to have a low degeneracy and hence a significant potential of predicting biological activity of certain classes of chemical compounds.

Results and Discussion

In this section, we discuss the light-harvesting dendrimer and Fréchet-type dendrimer and give close formulae of certain topological indices for these dendrimers. For background materials, see references[29,30].

Structure of light-harvesting dendrimer, $D_1[n]$, that we used in our study is depicted in Figure 1. Here n is the step of growth in the type of dendrimer.



(a) (b) (c)

Figure 1. a) The core of light-harvesting dendrimer $D_1[n]$; b) A branch of $D_1[1]$; c) A branch of $D_1[2]$.

Theorem 1. *The Zagreb polynomials of* $D_1[n]$ *are computed as follows:*

$$M_1(D_1[n],x) = (9\times 2^{n+1}+7)x^6 + (24\times 2^{n+1}+4)\,x^5 + (13\times 2^{n+1}+7)x^4,$$
$$M_2(D_1[n],x) = (9\times 2^{n+1}+7)x^9 + (24\times 2^{n+1}+4)x^6 + (13\times 2^{n+1}+7)x^4.$$

Proof. It is clear that $|V(D_1[n])| = 39\times 2^{n+1}+15$ and $|E(D_1[n])| = 46\times 2^{n+1}+18$. For computing the first and second Zagreb polynomials of $D_1[n]$, we partition the set of edges into three subsets, (a) edge E_1 with ended vertices of degree 2 and 2, (b) edge E_2 with ended vertices of degree 2 and 3, (c) edge E_3 with ended vertices of degree 3 and 3. It is easy to see that

$$|E_1(D_1[n])| = 13\times 2^{n+1}+7,$$
$$|E_2(D_1[n])| = 24\times 2^{n+1}+4,$$
$$|E_3(D_1[n])| = 9\times 2^{n+1}+7.$$

Now using equations (1) and (2), we have

$$M_1(D_1[n],x) = \sum_{uv\in E(D_1[n])} x^{(d_u+d_v)}$$

$$= \sum_{uv\in E_1} x^4 + \sum_{uv\in E_2} x^5 + \sum_{uv\in E_3} x^6$$

$$= (13\times 2^{n+1}+7)x^4 + (24\times 2^{n+1}+4)x^5$$
$$+ (9\times 2^{n+1}+7)x^6.$$

Also,

$$M_2(D_1[n],x) = \sum_{uv\in E(D_1[n])} x^{(d_u\times d_v)}$$

$$= \sum_{uv\in E_1} x^4 + \sum_{uv\in E_2} x^6 + \sum_{uv\in E_3} x^9$$

$$= (13\times 2^{n+1}+7)x^4 + (24\times 2^{n+1}+4)x^6$$
$$+ (9\times 2^{n+1}+7)x^9.$$

That proves our theorem.

The following corollary is immediate consequence of Theorem 1.

Corollary 1. *Let* $D_1[n]$ *be a dendrimer. Then*

$$M_1(D_1[n]) = 226\times 2^{n+1}+90,$$
$$M_2(D_1[n]) = 277\times 2^{n+1}+115.$$

Proof. We know the first and second Zagreb indices will be the first derivative of $M_1(D_1[n],x)$ and $M_2(D_1[n],x)$ evaluated at $x=1$, respectively. Thus,

$$M_1(D_1[n]) = \frac{\partial M_1(D_1[n],x)}{\partial x}|_{x=1}$$
$$= 6\times(9\times 2^{n+1}+7) + 5\times(24\times 2^{n+1}+4)$$
$$+4\times(13\times 2^{n+1}+7)$$
$$= 226\times 2^{n+1}+90.$$

Also,

$$M_2(D_1[n]) = \frac{\partial M_2(D_1[n], x)}{\partial x}|_{x=1}$$

$$= 9 \times (9 \times 2^{n+1} + 7) + 6 \times (24 \times 2^{n+1} + 4)$$

$$+4 \times (13 \times 2^{n+1} + 7)$$

$$= 277 \times 2^{n+1} + 115.$$

Which proves the theorem.

Theorem 2. *Let $D_1[n]$ be a dendrimer. The following topological indices can be calculated:*

$$PM_1(D_1[n]) = 4^{(13\times2^{n+1}+7)} \times 5^{(24\times2^{n+1}+4)} \times 6^{(9\times2^{n+1}+7)},$$

$$PM_2(D_1[n]) = 4^{(13\times2^{n+1}+7)} \times 6^{(24\times2^{n+1}+4)} \times 9^{(9\times2^{n+1}+7)},$$

$$HM(D_1[n]) = 1132 \times 2^{n+1} + 464.$$

Proof. By using edge partition given in Theorem 1 and equations (5)-(7), we have

$$PM_1(D_1[n]) = \prod_{uv\in E(D_1[n])} (d_u + d_v)$$

$$= \prod_{uv\in E_1} (d_u + d_v) \times \prod_{uv\in E_2} (d_u + d_v) \times \prod_{uv\in E_3} (d_u + d_v)$$

$$= 4^{|E_1(D_1[n])|} \times 5^{|E_2(D_1[n])|} \times 6^{|E_3(D_1[n])|}$$

$$= 4^{(13\times2^{n+1}+7)} \times 5^{(24\times2^{n+1}+4)} \times 6^{(9\times2^{n+1}+7)}.$$

$$PM_2(D_1[n]) = \prod_{uv\in E(D_1[n])} (d_u \times d_v)$$

$$= \prod_{uv\in E_1} (d_u \times d_v) \times \prod_{uv\in E_2} (d_u \times d_v) \times \prod_{uv\in E_3} (d_u \times d_v)$$

$$= 4^{|E_1(D_1[n])|} \times 6^{|E_2(D_1[n])|} \times 9^{|E_3(D_1[n])|}$$

$$= 4^{(13\times2^{n+1}+7)} \times 6^{(24\times2^{n+1}+4)} \times 9^{(9\times2^{n+1}+7)}.$$

$$
\begin{aligned}
HM(D_1[n]) &= \sum_{uv\in E(D_1[n])} (d_u + d_v)^2 \\
&= \sum_{uv\in E_1} (d_u + d_v)^2 + \sum_{uv\in E_2} (d_u + d_v)^2 + \sum_{uv\in E_3} (d_u + d_v)^2 \\
&= \sum_{uv\in E_1} 4^2 + \sum_{uv\in E_2} 5^2 + \sum_{uv\in E_3} 6^2 \\
&= 1132 \times 2^{n+1} + 464.
\end{aligned}
$$

Which finishes the proof of this theorem.

Theorem 3. *The total eccentricity polynomial of* $D_1[n]$ *is computed as follows:*

$$
\begin{aligned}
\theta(D_1[n], x) = {} & 2^n(4x^{10n+39} + 8x^{10n+38} + 10x^{10n+37} + 10x^{10n+36} \\
& + 8x^{10n+35} + 4x^{10n+34} + 4x^{10n+33} + 4x^{10n+32} \\
& + 4x^{10n+31} + 2x^{10n+30} + 2x^{10n+29}) \\
& + \sum_{k=1}^{n} 2^k\big(3x^{5(n+k)+28} + 2x^{5(n+k)+27} + 2x^{5(n+k)+26} \\
& + x^{5(n+k)+25} + x^{5(n+k)+24}\big) + 4x^{5n+28} + 4x^{5n+27} \\
& + 4x^{5n+26} + 2x^{5n+25} + 4x^{5n+24} + 4x^{5n+23} + 2x^{5n+22} \\
& + 5x^{5n+21} + 4x^{5n+20}.
\end{aligned}
$$

Proof. We are ready to compute the total eccentricity polynomial of dendrimer $D_1[n]$. To do this, we first draw the molecule by HyperChem and then compute the distance matrix of the molecular graph by TopoCluj[31]. Finally, we prepare a MATLAB program for computing the total eccentricity polynomial of graph. In following, we consider some exceptional cases for this class of dendrimers:

For $n = 1$ (see Figure 2), the total eccentricity polynomial is equal to:

$$\theta(D_1[1], x) = 8x^{49} + 16x^{48} + 20x^{47} + 20x^{46} + 16x^{45} + 8x^{44} + 8x^{43} + 8x^{42} + 8x^{41} + 4x^{40} + 4x^{39} + 6x^{38} + 4x^{37} + 4x^{36} + 2x^{35} + 2x^{34} + 4x^{33} + 4x^{32} + 4x^{31} + 2x^{30} + 4x^{29} + 4x^{28} + 2x^{27} + 5x^{26} + 4x^{25}.$$

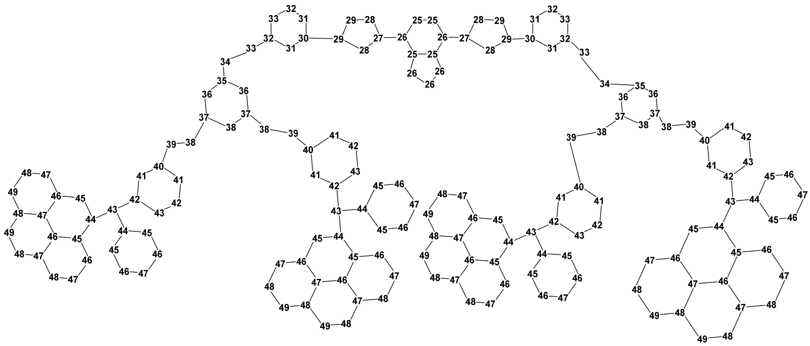


Figure 2. The eccentricity of vertices $D_1[n]$ with $n = 1$.

For $n = 2$ and 3, we have

$$\theta(D_1[2], x) = 16x^{59} + 32x^{58} + 40x^{57} + 40x^{56} + 32x^{55} + 16x^{54} + 16x^{53} + 16x^{52} + 16x^{51} + 8x^{50} + 8x^{49} + 12x^{48} + 8x^{47} + 8x^{46} + 4x^{45} + 4x^{44} + 6x^{43} + 4x^{42} + 4x^{41} + 2x^{40} + 2x^{39} + 4x^{38} + 4x^{37} + 4x^{36} + 2x^{35} + 4x^{34} + 4x^{33} + 2x^{32} + 5x^{31} + 4x^{30}.$$

$$\theta(D_1[3],x) = 32x^{69} + 64x^{68} + 80x^{67} + 80x^{66} + 64x^{65} + 32x^{64} + 32x^{63} + 32x^{62} + 32x^{61} + 16x^{60} + 16x^{59} + 24x^{58} + 16x^{57} + 16x^{56} + 8x^{55} + 8x^{54} + 12x^{53} + 8x^{52} + 8x^{51} + 4x^{50} + 4x^{49} + 6x^{48} + 4x^{47} + 4x^{46} + 2x^{45} + 2x^{44} + 4x^{43} + 4x^{42} + 4x^{41} + 2x^{40} + 4x^{39} + 4x^{38} + 2x^{37} + 5x^{36} + 4x^{35}.$$

We find the eccentric partition of $D_1[n]$ based on the largest distance between u and any other vertex v. Therefore, one can see that there are twenty five types of vertices of dendrimer graph $D_1[n]$. Table 1 explains such partition for $D_1[n]$.

It is easy to check that:

$\forall u \in V(D_1[n])$;

$Max\left(\varepsilon_{D_1[n]}(u)\right) = 10n + 39$ and $Min\left(\varepsilon_{D_1[n]}(u)\right) = 5n + 20$.

Table 1. Eccentricity of all vertices of $D_1[n]$.

Eccentricity	**No. vertices**	**Eccentricity**	**No. vertices**	**Eccentricity**	**No. vertices**
$10n + 39$	2^{n+2}	$10n + 30$	2^{n+1}	$5n + 26$	4
$10n + 38$	2^{n+3}	$10n + 29$	2^{n+1}	$5n + 25$	2
$10n + 37$	10×2^n	$5(n + k) + 28$	$3\sum_{k=1}^{n} 2^k$	$5n + 24$	4
$10n + 36$	10×2^n	$5(n + k) + 27$	$\sum_{k=1}^{n} 2^{k+1}$	$5n + 23$	4
$10n + 35$	2^{n+3}	$5(n + k) + 26$	$\sum_{k=1}^{n} 2^{k+1}$	$5n + 22$	2
$10n + 34$	2^{n+2}	$5(n + k) + 25$	$\sum_{k=1}^{n} 2^k$	$5n + 21$	5

Table 1. Continued

$10n+33$	2^{n+2}	$5(n+k)+24$	$\sum_{k=1}^{n} 2^k$	$5n+20$	4
$10n+32$	2^{n+2}	$5n+28$	4		
$10n+31$	2^{n+2}	$5n+27$	4		

By using data in Table 1 and equation (8) the proof of is straightforward.

Theorem 4. *The eccentric connectivity polynomial of* $D_1[n]$ *is computed as follows:*

$$\begin{aligned}\xi^c(D_1[n],x) = 2^n(&8x^{10n+39} + 18x^{10n+38} + 24x^{10n+37} + 24x^{10n+36} \\ &+ 18x^{10n+35} + 12x^{10n+34} + 10x^{10n+33} + 10x^{10n+32} \\ &+ 8x^{10n+31} + 6x^{10n+30} + 4x^{10n+29}) \\ &+ \sum_{k=1}^{n} 2^k \left(6x^{5(n+k)+28} + 6x^{5(n+k)+27} + 4x^{5(n+k)+26}\right. \\ &\left.+ 3x^{5(n+k)+25} + 2x^{5(n+k)+24}\right) + 8x^{5n+28} + 10x^{5n+27} \\ &+ 8x^{5n+26} + 6x^{5n+25} + 10x^{5n+24} + 8x^{5n+23} + 6x^{5n+22} \\ &+ 12x^{5n+21} + 10x^{5n+20}.\end{aligned}$$

Proof. Using Theorem 3 and equation (9), we are done.

The proof of the following corollary follows easily from Theorem 3 and Theorem 4, hence it is left to the reader.

Corollary 2. *The total eccentricity index and eccentric connectivity index of* $D_1[n]$ *are computed as follows:*

$$\theta(D_1[n]) = 75n + 2^n(780n + 2496) + 403,$$

$$\xi^c(D_1[n]) = 180n + 2^n(1840n + 5884) + 956.$$

We now consider the second type of dendrimer namely $D_2[n]$, see Figure 3. In fact, Fréchet-type dendrimers are based on polybenzyl ether hyper branched skeleton.

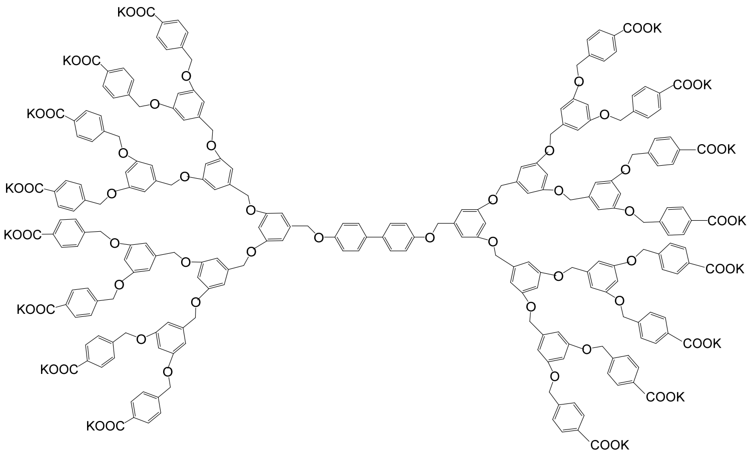


Figure 3. Fréchet-type dendrimer $D_2[n]$ for $n = 3$.

Theorem 5. *Consider the graph* $D_2[n]$ *of a Fréchet-type dendrimer. The following topological indices can be calculated:*

$$M_1(D_2[n]) = 200 \times 2^n - 26,$$
$$M_2(D_2[n]) = 115 \times 2^{n+1} - 31,$$
$$HM(D_2[n]) = 964 \times 2^n - 132,$$
$$PM_1(D_2[n]) = 4^{2^{n+2}} \times 4^{(2^{n+3}+2)} \times 5^{(7\times 2^{n+2}-8)} \times 6^{(2^{n+1}+1)},$$
$$PM_2(D_2[n]) = 3^{2^{n+2}} \times 4^{(2^{n+3}+2)} \times 6^{(7\times 2^{n+2}-8)} \times 9^{(2^{n+1}+1)}.$$

Proof. Let $D_2[n]$ be the graph of Fréchet-type dendrimer. We have $|V(D_2[n])| = 19 \times 2^{n+1} - 4$ and $|E(D_2[n])| = 21 \times 2^{n+1} - 5$. For computing these topological indices of $D_2[n]$, we partition the set of edges into four subsets, (a) edge E_1 with ended vertices of degree 1 and 3, (b) edge E_2 with ended vertices of degree 2 and 2, (c) edge E_3 with ended vertices of degree 2 and 3, (d) edge E_4 with ended vertices of degree 3 and 3. It is easy to see that

$$|E_1(D_2[n])| = 2^{n+2},$$
$$|E_2(D_2[n])| = 2^{n+3} + 2,$$
$$|E_3(D_2[n])| = 7 \times 2^{n+2} - 8,$$
$$|E_4(D_2[n])| = 2^{n+1} + 1.$$

By using edge partition given and equations (3)-(7), we are done.

Theorem 6. *The total eccentricity polynomial of the dendrimer* $D_2[n]$ *is computed as follows:*

$$\theta(D_2[n], x) = 2^{n+1}(2x^{10n+23} + x^{10n+22} + x^{10n+21} + 2x^{10n+20}) + \sum_{k=1}^{n} 2^k \left(4x^{5(n+k)+19} + 2x^{5(n+k)+18} + 2x^{5(n+k)+17} + 3x^{5(n+k)+16} + 2x^{5(n+k)+15}\right) + 4x^{5n+19} + 2x^{5n+18} + 2x^{5n+17} + 2x^{5n+16} + 2x^{5n+15} + 4x^{5n+14} + 4x^{5n+13} + 2x^{5n+12}.$$

Proof. The proof is similar to the proof of Theorem 1. Let $D_2[n]$ be the graph of Fréchet-type dendrimer. Since

$$\theta(D_2[n], x) = \sum_{u \in V(D_2[n])} x^{\varepsilon_{D_2[n]}(u)},$$

for $n = 1$ (see Figure 4), the total eccentricity polynomial is equal to:

$$\theta(D_2[1],x) = 8x^{33} + 4x^{32} + 4x^{31} + 8x^{30} + 8x^{29} + 4x^{28} + 4x^{27} + 6x^{26} + 4x^{25} + 4x^{24} + 2x^{23} + 2x^{22} + 2x^{21} + 2x^{20} + 4x^{19} + 4x^{18} + 2x^{17}.$$

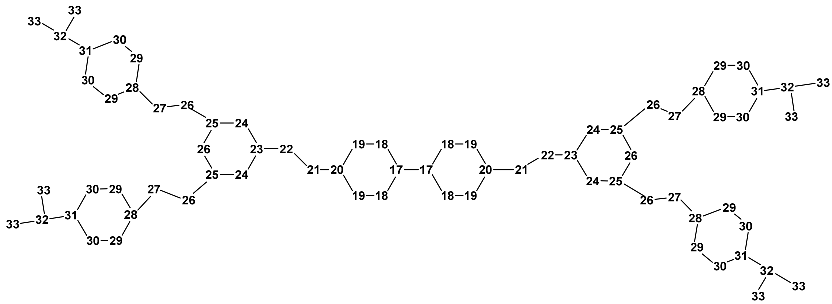


Figure 4. The eccentricity of vertices of $D_2[n]$ for $n = 1$.

For $n = 2$ and 3, we have

$$\theta(D_2[2],x) = 16x^{43} + 8x^{42} + 8x^{41} + 16x^{40} + 16x^{39} + 8x^{38} + 8x^{37} + 12x^{36} + 8x^{35} + 8x^{34} + 4x^{33} + 4x^{32} + 6x^{31} + 4x^{30} + 4x^{29} + 2x^{28} + 2x^{27} + 2x^{26} + 2x^{25} + 4x^{24} + 4x^{23} + 2x^{22}.$$

$$\theta(D_2[3],x) = 32x^{53} + 16x^{52} + 16x^{51} + 32x^{50} + 32x^{49} + 16x^{48} + 16x^{47} + 24x^{46} + 16x^{45} + 16x^{44} + 8x^{43} + 8x^{42} + 12x^{41} + 8x^{40} + 8x^{39} + 4x^{38} + 4x^{37} + 6x^{36} + 4x^{35} + 4x^{34} + 2x^{33} + 2x^{32} + 2x^{31} + 2x^{30} + 4x^{29} + 4x^{28} + 2x^{27}.$$

We find the eccentric partition of $D_2[n]$ based on the largest distance between u and any other vertex v. Therefore, one can see that there are seventeen types of vertices of dendrimer graph $D_2[n]$. Table 2 explains such partition for $D_2[n]$.

It is easy to check that:

$\forall u \in V(D_2[n])$;

$Max\left(\varepsilon_{D_2[n]}(u)\right) = 10n + 23$ and $Min\left(\varepsilon_{D_2[n]}(u)\right) = 5n + 12$.

Table 2. Eccentricity of all vertices of $D_2[n]$.

Eccentricity	No. vertices	Eccentricity	No. vertices	Eccentricity	No. vertices
$10n + 23$	2^{n+2}	$5(n+k) + 17$	$\sum_{k=1}^{n} 2^{k+1}$	$5n + 16$	2
$10n + 22$	2^{n+1}	$5(n+k) + 16$	$3\sum_{k=1}^{n} 2^{k}$	$5n + 15$	2
$10n + 21$	2^{n+1}	$5(n+k) + 15$	$\sum_{k=1}^{n} 2^{k+1}$	$5n + 14$	4
$10n + 20$	2^{n+2}	$5n + 19$	4	$5n + 13$	4
$5(n+k) + 19$	$\sum_{k=1}^{n} 2^{k+2}$	$5n + 18$	2	$5n + 12$	2
$5(n+k) + 18$	$\sum_{k=1}^{n} 2^{k+1}$	$5n + 17$	2		

Now by using the partition given in Table 2 and equation (8) the proof of is straightforward.

Theorem 7. *The eccentric connectivity polynomial of the dendrimer* $D_2[n]$ *is computed as follows:*

$$\xi^c(D_2[n],x) = 2^{n+1}(2x^{10n+23} + 3x^{10n+22} + 3x^{10n+21} + 4x^{10n+20}) + \sum_{k=1}^{n} 2^k \big(8x^{5(n+k)+19} + 6x^{5(n+k)+18} + 4x^{5(n+k)+17} + 6x^{5(n+k)+16} + 6x^{5(n+k)+15}\big) + 8x^{5n+19} + 6x^{5n+18} + 4x^{5n+17} + 4x^{5n+16} + 6x^{5n+15} + 8x^{5n+14} + 8x^{5n+13} + 6x^{5n+12}.$$

Proof. The proof can obtain by using of Table 2 and equation (9).

The proof of the following corollary follows easily from Theorem 6 and Theorem 7, hence it is left to the reader.

Corollary 3. *The total eccentricity index and eccentric connectivity index of $D_2[n]$ are computed as follows:*

$$\theta(D_2[n]) = 2^n(380n + 576) - 20n + 22,$$
$$\xi^c(D_2[n]) = 2^n(840n + 1238) - 50n + 42.$$

Conclusions

In this paper by means of edge dividing approaches, we derived closed formulae of several important topological indices for light-harvesting dendrimer and Fréchet-type dendrimer, including first Zagreb index, second Zagreb index, first multiple Zagreb index, second multiple Zagreb index and hyper Zagreb index. In addition, two distance based topological indices, the total eccentricity index and eccentric connectivity index of these dendrimers were calculated. The results obtained in our paper illustrate the promising application prospects in chemical and pharmacy engineering.

Acknowledgements

The authors would like to thank the anonymous referee for his/her helpful comments that have improved the presentation of results in this article.

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